Mount Kenya University

UNIVERSITY EXAMINATION 2014/2015

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

BACHELOR OF EDUCATIOB ARTS
SCHOOL BASED

UNIT CODE: BMA1202 UNIT TITLE: DISCRETE MATHEMATICS

DATE: AUGUST 2015 MAIN EXAM TIME: 2 HOURS

Instructions: Answer question one and any other two

1. a) Differentiate combination and permutation. (2 Marks)

   b) Show that $\neg P \vee \neg q \equiv \neg(p \wedge q)$ using truth tables. (4 Marks)

   c) Draw the graph corresponding to the adjacency matrix below

$$A = \begin{pmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

(4 Marks)

   d) Determine the power set P(x) of x={1,2,3} (3 Marks)

   e) Prove by induction $1^2 + 3^2 + 5^2 + \ldots (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

(6 Marks)

   f) Draw the truth for the Boolean expression $Z = A.B + \overline{A}.\overline{B}$

(4 Marks)

g) Twenty four dogs are in a kernel. Twelve of the dogs are black, six of the dogs have short tails and fifteen of the dogs have long hair. There is only one dog that is black, with short tail and long hair. Two of the dogs have short tail with long hair but are not black. If all of the dogs of the dogs in a kernel have at least one of these characteristics, how many dogs are black with long hair but do not have short tails? (9 Marks)

2. a) Three Boolean variables x_1, x_2 and x_3 are such that $A = x_1 + x_2$, $B = x_1.x_3$ and $C = (x_1 + x_2).x_3$. Fill in the table below. (5 Marks)

X1	X2	X3	A	B	C	A.B	A+B+C
1	1	1					
1	1	0					
1	0	1					
1	0	0					
0	1	1					
0	1	0					
0	0	1					
0	0	0					

b) Show that $A \vee (A \wedge B)$ is a tautology if A and B are propositions. (4 Marks)

c) The dean has announced that: 'If mathematics department get additional $400000, then it will hire a new faculty member'

i) Define two propositions from the deans statement. (2 Marks)

ii) Construct a truth table for all combinations of conditional propositions. (4 Marks)

iii) If the mathematics department doesn't get additional 40000 and the mathematics department hires the new faculty member, what is the truth value of the deans statement. (1 Mark)

d) Show that the proposition $p \vee (p \wedge q)$ is a tautology. (4 Marks)

3. a) Define the following;
   i) Power set (2 Marks)
   ii) Recursive definitions of sets (2 Marks)
   iii) Relative complement of sets. (2 Marks)

   b) Given $U=\{x/:1\langle x10$ x is an integer, A is a set of odd numbers, B is a set of factors of 24, C={3,10}
   i) List all the elements for each set. (3 Marks)

   ii) Draw the venn diagram to show the relationship between U, A, B and C. (3 Marks)

   iii) Using the venn diagram or otherwise, find the set
   - a) $(A \cup B)'$ (1 Mark)
   - b) $(A \cup C)'$ (1 Mark)
   - c) $(A \cup B \cup C)'$ (1 Mark)

   c) Given the following proposition
   p: 1+1=3
   q: 2 divide 4
   r: LCM of 6 and 4 is 2
   Find the values of the following compound proposition:
   i) $p \wedge q$
   ii) $p \wedge q \vee \overline{r}$
   iii) $\overline{p} \wedge q \rightarrow r$ (5 Marks)

4. a) Nine players are available to play a tennis team of players. In how many was the team can be selected if 2 players are brothers and must both be included in the team. (4 Marks)

   b) Use mathematical induction to prove the formula $3+6+9+,,,3n=\frac{3n(n+1)}{2}$ if n is any natural number. (5 Marks)

c) How many ways can 10 mathematics books and 7 physics book be shelved if mathematics and 2 physics books are picked each time.

(6 Marks)

d) Some group behavior studies investigate the influence one person has on another in some social setting. Jose influenced Bob, Dan, Joan and Louse influenced Jose, Dan and Joan. Draw a directed graph to show the influence relationship hence write the adjacency matrix to represent this graph. (5 Marks)

5. a) Of 40 boys in a form, 32 plays hockey 2 play tennis and 8 play football. Every boy plays at least one game and 4 plays all the games. How many play two and only two games? (8 Marks)

b) Write the dual of the following Boolean algebra.

i) x+y=1

ii) X.y=0 (2 Marks)

c) Evaluate;

i) $15P_5$ (2 Marks)

ii) $15C_5$ (3 Marks)

d) Show that if p and q are proposition then $\overline{p \vee r}$ and $\bar{p} \wedge \bar{q}$ are logically equivalent using truth table. (5 Marks)

QUESTION 1(b)

Show that if p and q are propositions then $\overline{pvq}$ and $\overline{p \wedge q}$ logically equivalent using the truth table. (4 Marks)

QUESTION 2(d)

Show that the proposition $p^{\vee} -(p^{\wedge}q)$ is a tautology. (4 Marks)