

# Mount Kenya University



UNIVERSITY EXAMINATION 2014/2015

SCHOOL OF PURE AND APPLIED SCIENCES  
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

BEDA/BEDSCI/BSNE  
SCHOOL BASED

UNIT CODE: BMA2101 UNIT TITLE: CALCULUS III

DATE: AUGUST 2015

MAIN EXAM

TIME: 2 HOURS

Instructions: Answer question one and any other two

1. a) Sketch the graph of the function  $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$  and state its domain and range. (3 Marks)
- b) Investigate the converges of the following series using the ratio test  $\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$  (4 Marks)
- c) Find the maclaurin's series generated by  $f(x) = \cos x$  (4 Marks)
- d) Express  $\frac{dw}{dt}$  using the chain rule as a function of t and evaluate  $\frac{dw}{dt}$  at given value if  $w(x, y) = x^2 + y^2 + z$  and  $x = \cos t, y = \sin t, z = t$  and (4 Marks)
- e) Find and classify the critical points of  $f(x, y) = x^3 + y - 3xy + 4$  (10 Marks)

f) Find the value of a number C in the conclusion of the mean value theorem that satisfies the equation  $\frac{f(b)-f(a)}{b-a} = f'(c)$  in the interval

$$f(x) = 5x^2 - 3x + 1 \quad [1, 3]$$

(5 Marks)

2. a) Evaluate  $\int_c F \cdot dr$  where  $F(x, y, z) = zi + xi - y^2k$  along the curve c given by  $r(t) = t^2 + tj + \sqrt{t}k, 0 < t < 1$

(5 Marks)

b) Find the work done by conservative force field  $f = yzi + xzj + xyk$  where  $F = 2xi + 3yj + 4zk$  along a smooth curve A(1, 3, 9) to B(1, 6, 4)

(5 Marks)

c) Show that F is conservative force field and calculate its potential function f if

(10 Marks)

$$F = (2xy - 3yz)\mathbf{i} + (x^2 - 3xz)\mathbf{j} + (6z^2 - 3xy)\mathbf{k}$$

3. a) Find absolute maxima and minima values of  $f(x, y) = 2 + 2x + 2y - x^2 - y^2$  on the rectangular region in the first quadrant bounded by lines  $x=0$ ,  $y=0$  and  $y=9-x$ .

(10 Marks)

b) Find the volume of the prism whose base is a triangle in the xy plane bounded by the x axis and the line  $y=x$  and  $x=1$  and the top lies in the plane

(6 Marks)

c) Find the Taylor's series generated by  $f(x) = e^{2x}$  near  $x=0$

(4 Marks)

4. a) Use the Stoke's theorem to evaluate  $\oint_c F \cdot dr$  if  $F = xzi + xyj + 3xzk$  and c is the boundary of the portion of the plane in the first octane, transverse counterclockwise as viewed from above.

(10 Marks)

b) Find  $f_x, f_y, f_{xy}, f_{xx}$  and  $f_{yy}$  if  $f(x, y) = x^2y + 3xy + 4y^2 + 3$  and evaluate these partial derivatives at point (2, 2)

(5 Marks)

c) Given that  $1 - \frac{x^2}{4} \leq u(x) \leq 1 + \frac{x^2}{2}$  for  $x \neq 0$  estimate the  $\lim_{x \rightarrow 0} u(x)$  (5 Marks)

5. a) i) State the Fourier series (2 Marks)

ii) Obtain the four series for the periodic function  $f$  define as;

$$f(x) = \begin{cases} -4, & \text{when } -\pi < x < 0 \\ 4, & \text{when } 0 < x < \pi \end{cases} \quad (8 \text{ Marks})$$

b) Evaluate the triple integral  $\iiint (x^2 + y^2 + z^2) dz dy dx$  for  $0 < z < 1, 0 < y < 1, 0 < x < 1$  (5 Marks)

c) Given that  $z = \frac{2xy}{y + \cos x}$  find  $\frac{dz}{dx}$  and  $\frac{dz}{dy}$  (5 Marks)