KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2008/2009

SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR SCIENCE

SMA 201: CALCULUS III

DATE: MONDAY 6TH APRIL 2009 TIME: 11.00 A.M. – 1.00 P.M.

INSTRUCTIONS

Answer Question One and any other two questions

QUESTION ONE (30 MARKS)

a) Find $\lim_{x \to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$ (3 marks)

b) Obtain the first three nonzero terms in the Maclaurin series for

$$f(x) = \frac{x}{1-x}$$ (4 marks)

c) Find the value of c guaranteed by Cauchy Mean Value Theorem if $f(x) = \frac{x^3}{x} - 4x$ and $g(x) = x^2$ defined on [0,3].

(4 marks)

d) Find $\frac{\partial w}{\partial x}$ at the point (2, -1, 1) if

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ii) normal line to surface at point $(0,1,2)$.

(5 marks)

QUESTION FOUR (20 MARKS)

a) Locate all the critical points on the graph of $f(x,y)=8x^3-24xy+y^3$ and use the second partial derivative test to classify each point as a relative extremum or saddle point. (9 marks)

b) Find the direction in which the function $f(x,y,z)=\ln(x^2+y^2-1)+y+6z$ increases and decreases most rapidly at point $(1,1,0)$. At what rate does f change in these directions? (7 marks)

c) If $f(x,y)=x^3y+e^{xy^2}$, find f_{xx} and f_{xy}. (4 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate the double integral $\iint_R (x^2+y^2)\,dA$ where R is the region in the xy-plane bounded $y=x^2$, $x=2$ and $y=1$. (6 marks)

b) Apply the Green's Theorem to evaluate the line integral $\oint_c (2xy^3\,dx+4x^2y^2\,dy)$ where, c is the boundary of the triangular region in the first quadrant enclosed by the x-axis, the line x = 1 and the curve $y = x^3$ (7 marks)

c) Evaluate $\iint_s (\nabla\times F)\bullet n\,ds$ using stokes Theorem where S is the portion of $z=4-x^2-y^2$ above the xy – plane and the field $F=xzi+2yj+z^3k$.

(7 marks)

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$W = x^2 + y^2 + z^2, \; z^3 - xy + yz + y^3 = 0$ and where x and y are the independent variables. (4 marks)

e) Obtain the equation of the tangent plane of the curve $x^2 + y^2 - 2xy - x + 3y - z = -4$ at a point (2, -3, 18). (4 marks)

f) Find the Fourier transform of the function

$$f(x) = \begin{cases} 1; & 0 < x < \pi \\ 0; & \pi < x < 2\pi \\ f; & f(x + 2\pi) \end{cases}$$

(3 marks)

g) Evaluate $\iint_R f(x,y)\,dxdy$ where $f(x,y) = x + 2y$ and R is the region defined by $1 \le x \le 4$ and $1 \le y \le 2$. (4 marks)

i) Use Green's theorem in the plane to evaluate $\oint\left(x^2 + xy^3\right)dx + \left(y^2 - 2xy\right)dy$ where c is a square with vertices (0,0), (2,0), (2,2) and (0,2). (4 marks)

QUESTION TWO (20 MARKS)

a) i) State the Mean Value Theorem. (3 marks)

ii) The function $y = |1 - x^2|$ defined on $-2 \le x \le 2$, has a horizontal tangent at $x = 0$ even though the function is not differentiable at $x = -1$ and $x = 1$. Does this contradict the Mean Value Theorem? Explain. (4 marks)

b) Suppose you know that f(x) is differentiable and the $f'(x)$ always has a value between -1 and +1. Show that $|f(x) - f(a)| \le |x - a|$ (3 marks)

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QUESTION FOUR (20 MARKS)

a) Show that if $f(u,v,w)$ and its partial derivatives w.r.t. v and w are continuous and that

$$u = x - y, \quad v = y - z \quad and \quad w = z - x, \quad then$$

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$$

[5 marks]

b) Evaluate $\int_0^1\int_0^1\int_0^1 (x^2 + y^2 + z^2)\, dz\, dy\, dx$

[5 marks]

c) Let $f(x) = x^2 y + \cos y + y \sin x$

Find $\frac{\partial^3 f}{\partial x^2 \partial y}$ [4 marks]

d) Find the equation of the tangent plane and the normal line of the curve $\cos \pi x - x^2 y + e^{xz} + yz = 4$ at the point $(0,1,2)$. [6 marks]

QUESTION FIVE (20 MARKS)

a) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition.

$x + 2y + 4z = 42$ [6 marks]

b) Find the direction in which $f(x,y,z) = (x+y)^2 + (y+z)^2 + (z+x)^2$ increases most rapidly at $(1,1,1)$ and find the rate at which f changes in these directions.

[5 marks]

c) Evaluate $\int_0^1\int_0^{1-x}\int_{4x^2}^{5-x^2} (x - y + 1)\, dz\, dy\, dx$ [5 marks]

(d) Verify Greens theorem in the plane for $\oint_c y(2xy-1)dx + x(2xy+1)dy$ where c is the circle $x^2 + y^2 = 36$. [4 marks]

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