

Mount Kenya



University

UNIVERSITY EXAMINATION 2014/2015

SCHOOL OF PURE AND APPLIED SCIENCES  
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

BACHELOR OF EDUCATION  
SCHOOL BASED

UNIT CODE: BMA2101

UNIT TITLE: CALCULUS III

DATE: APRIL/MAY 2015

MAIN EXAM

TIME: 2 HOURS

**INSTRUCTIONS:** Answer question one and any other two

1. a) Find the volume of a region bounded above by the parabola  $z=x^2+y^2$  and below by a square R on xy-plane such that  $R: -1 \leq x \leq 1, -1 \leq y \leq 1$  (4 Marks)
- b) Find the graph of  $f(x)$  and state its domain and range if;  $f(x) = \frac{1}{x^2}$  (3 Marks)
- c) Use the L'hospital's rule to evaluate  $\lim_{x \rightarrow 0} \frac{8x^2}{\cos x - 1}$  (3 Marks)
- d) Find the taylor series generated by  $f(x) = \cos x$  at  $x = \frac{\pi}{2}$  (4 Marks)
- e) Given that  $f(x,y) = x^3y^4 + x^2 - 3y^2$ , find  $f_{xy}$  and  $f_{yx}$ . (4 Marks)
- f) Determine all the numbers C which satisfy the conclusion of the mean value theorem for the function  $f(x) = x^3 + 2x^2 - x$  on  $[-1, 2]$  where mean value theorem states;
- g) Evaluate  $f'(c) = \frac{f(b) - f(a)}{b - a}$  the limit of  $f(x) = \frac{x^2 + x - 2}{x^2 - x}$  as x tends to 1. (4 Marks)

h) Suppose that the temperature  $T$  at each point  $(x,y,z)$  in a region of space is given by  $T=100-x^2-y^2-z^2$  and that  $F(x,y,z)$  is defined to be gradient of  $T$ . Find the vector field  $F$  (4 Marks)

2. a) Find the Taylor series generated by  $f(x) = \frac{1}{x}$  at  $x=2$ . Does the series

converge to  $\frac{1}{x}$  (6 Marks)

b) Find  $f_x, f_y$  and  $f_z$  given that  $f(x,y,z)=(x^2+1)(y+z)$  (4 Marks)

c) Find the maclaurin's series generated by  $\cos x$ . (4 Marks)

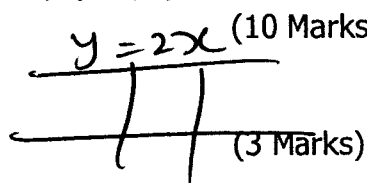
d) Evaluate the double integrals over the given regions

i)  $\iint_R (6y^2 - 2x) dA : 0 \leq x \leq 1, 0 \leq y \leq 2$

ii)  $\iint_R (6y^2 - 2x) dA : 0 \leq x \leq 1, 0 \leq y \leq 2$  (6 Marks)

3. a) Find the absolute maxima and minima of the function on the given domain  $f(x)=2x^2-4x+y^2+1$  on closed triangle bounded by  $x=0, y=2, y=2x$  in the first quadrant. (10 Marks)

b) Evaluate  $\int_1^2 \int_0^4 2xy ddx$



c) Write down the first few terms of the series to show how the series starts then

find the sum of the series  $\sum \frac{(-1)^n 5}{4^n}$  (3 Marks)

d) Find the fourier series associated with the function  $f(x)=1$   $0 < x < 2\pi$  (4 Marks)

4. a) Use the Stokes theorem to evaluate  $\oint_C F \cdot dr$  if  $F = xzi + xyj + 3x^2k$  and C is the boundary of the portion of the plane  $2x+y+z=2$  in the first octant, traversed counter clockwise as viewed from above. (10 Marks)

b) Show that  $F = e^x \cos y + yz) + (xz - e^x \sin y) j + (xy + z)k$  is conservative field over its natural domain and find a potential function for it. (7 Marks)

c) Use the L'Hopital rule to evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^3}$  (3 Marks)

5. a) Use the chain rule to evaluate the derivative of  $w(x,y)=xy$ , with respect to t along a path given parametrically by  $x(t)=\cos t$ ,  $y(t)=\sin t$  at  $t=0$ . (4 Marks)

b) Find the Fourier series expansion of the function  $f(x) = \begin{cases} 1, & \text{if } 0 < x < \pi \\ 2, & \text{if } \pi < x \leq 2\pi \end{cases}$  (10 Marks)

c) Compute the given partial derivative of the following functions at the space field points (6 Marks)

i)  $f(x,y) = 1 - x + y - 3xy$ ,  $f_x$  and  $f_y$  at  $(1,2)$

ii)  $f(x,y) = \sqrt{x^2 + 3} - 1$ ,  $f_x$  and  $f_y$  at  $(-2,3)$

iii)  $f(x,y) = 4 - 2x - 3y - xy^2$ ,  $f_x$  and  $f_y$  at  $(-2,1)$