Mount Kenya University

UNIVERSITY EXAMINATION 2014/2015

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

BACHELOR OF EDUCATION
SCHOOL BASED

UNIT CODE: BMA2101 **UNIT TITLE: CALCULUS III**

DATE: APRIL/MAY 2015 **MAIN EXAM** **TIME: 2 HOURS**

INSTRUCTIONS: Answer question one and any other two

1. a) Find the volume of a region bounded above by the parabola $z=x^2+y^2$ and below by a square R on xy-plane such that R:$-1\leq x\leq 1$, $-1\leq y\leq 1$ (4 Marks)

   b) Find the graph of f(x) and state its domain and range if; $f(x)=\frac{1}{x^2}$ (3 Marks)

   c) Use the L'hopital's rule to evaluate $\lim_{x\to 0}\frac{8x^2}{\cos x-1}$ (3 Marks)

   d) Find the taylor series generated by f(x)=cos x at $x={}^{\pi}/_{2}$ (4 Marks)

   e) Given that $f(x,y)=x^3y^4+x^2-3y^2$, find f_{xy} and f_{yx}. (4 Marks)

   f) Determine all the numbers C which satisfy the conclusion of the mean value theorem for the function $f(x)=x^3+2x^2-x$ on [-1,2] where mean value theorem states;

   g) Evaluate $f'(c)=\frac{f(b)-f(a)}{b-a}$ the limit of $f(x)=\frac{x^2+x-2}{x^2-x}$ as x tends to 1. (4 Marks)

h) Suppose that the temperature T at each point (x,y,z) in a region of space is given by $T=100-x^2-y^2-z^2$ and that F(x,y,z) is defined to be gradient of T. Find the vector field F (4 Marks)

2. a) Find the Taylor series generated by $f(x)=\frac{1}{x}$ at x=2. Does the series converse to $\frac{1}{x}$ (6 Marks)

b) Find f_x, f_y and f_z given that $f(x,y,z)=(x^2+1)(y+z)$ (4 Marks)

c) Find the maclaurin's series generated by cosx. (4 Marks)

d) Evaluate the double integrals over the given regions

i) $\iint_R (6y^2-2x)dA R: 0 \leq x \leq 1, 0 \leq y \leq 2$

ii) $\iint_R (6y^2-2x)dA R: 0 \leq x \leq 1, 0 \leq y \leq 2$ (6 Marks)

3. a) Find the absolute maxima and minima of the function on the given domain $f(x)=2x^2-4x+y^2+1$ on closed triangle bounded by x=0, y=2, y=2x in the first quadrant. (10 Marks)

y = 2x

b) Evaluate $\int_1^2 \int_0^4 2xyddx$ (3 Marks)

c) Write down the first few terms of the series to show how the series starts then find the sum of the series $\sum \frac{(-1)^n 5}{4^n}$ (3 Marks)

d) Find the fourier series associated with the function f(x)=1 0<x<2п (4 Marks)

4. a) Use the stokes theorem to evaluate $\oint_c F.dr$ if $F = xzi + xyj + 3x^2k$ and C is the boundary of the portion of the plane 2x+y+z=2 in the first octant, transversed counter clockwise as viewed from above. (10 Marks)

b) Show that $F = e^x \cos y + yz) + (xz - e^x \sin y)\, \mathrm{j} + (\mathrm{xy} + \mathrm{z})\, \mathrm{k}$ is conservative field over its natural domain and find a potential function for it. (7 Marks)

c) Use the L'Hopital rule to evaluate $\lim_{x \to 0} it \dfrac{1 - \cos x}{x^3}$ (3 Marks)

5. a) Use the chain rule to evaluate the derivative of w(x,y)=xy, with respect to t along a path given parametrically by x(t)=cost, y(t)=sint at t=0. (4 Marks)

b) Find the fourier series expansion of the function $f(x) = \begin{cases} 1, if\, 0 \langle x \langle \pi \\ 2, if\, \pi \langle x \leq 2\pi \end{cases}$

(10 Marks)

c) Compute the given partial derivative of the following functions at the space field points (6 Marks)

i) $f(xy) = 1 - x + y - 3xy, f_x and f_y at (1,2)$

ii) $f(xy) = \sqrt{x^2 + 3 - 1}, f_x and f_y at (-2,3)$

iii) $\mathrm{f(x, y) = 4 - 2x - 3y - xy^2}\ f_x and f_y at (-2,1)$