KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

SMA 200: CALCULUS II

DATE: Tuesday, 29th November, 2011 TIME: 8.00 a.m. – 10.00 a.m.

INSTRUCTIONS:

Answer question **ONE** and any other **TWO.**

QUESTION ONE (30 MARKS)

a) Evaluate the following indefinite integrals

i) $\int \frac{1}{50^x}\,dx$ (2 marks)

ii) $\int x^2 \tan^{-1} x\,dx$ (4 marks)

iii) $\int \cos\theta \cos 4\theta\, d\theta$ (3 marks)

b) Evaluate the following definite integrals

i) $\int_0^{\pi} \frac{1}{5+3\cos\theta}\,d\theta$ (3 marks)

ii) $\int_1^{2} \frac{2x-5}{x^2+2x+2}\,dx$ (3 marks)

iii) $\int_0^{\pi} Sin^5 2x \cos^2 2x\,dx$ (3 marks)

c) Use fundamental theorem of calculus to find $\frac{d^2}{dx^2}\left(\int_{-x}^{x}\sqrt{t^6+4}\,dt\right)$ (4 marks)

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d) Find the volume of the solid generated when the region bounded by $y^2 = 4x$ *and* $y = 2x - 4$ is rotated about the y – axis. (4 marks)

e) Find the area bounded by the curves $y = 3x^2 - 2x$ *and* $y = 1 - 4x$. (4 marks)

QUESTION TWO (20 MARKS)

a) The area of the region defined by the inequalities $x^2 \leq y,\ y \geq x$ is rotated about the x – axis. Find the volume of the solid generated. (6 marks)

b) Find the area of the surface generated when the curve $x^2 = 4y$ between $y = 0$ *and* $y = 3$ is rotated about the y – axis. (7 marks)

c) A curve has equation $8y = -\dfrac{2}{x^2} - x^4$. Show that $\sqrt{1+\left(\dfrac{dy}{dx}\right)^2} = \dfrac{1}{2}\left(\dfrac{1}{x^2} + x^3\right)$.

Hence find the length of the curve from $x = 1$ *to* $x = 4$. (7 marks)

QUESTION THREE (20 MARKS)

a) Given that $I_n = \int \tan^{2n} x\, dx$, show that

$$I_n = \frac{\tan^{2n-1} x}{(2n-1)} - I_{n-1},\ n \geq 1.\ \text{Hence evaluate} \int \tan^6 x dx.$$

(10 marks)

b) Evaluate the following Improper Integrals

i) $\int_0^1 x\, In\, x\, dx$ (5 marks)

ii) $\int_1^2 \dfrac{dx}{x\sqrt{x-1}}$ (5 marks)

QUESTION FOUR (20 MARKS)

a) A rectangular swimming pool is 30 ft wide and 50 ft long. The table shows the depths $h(x)$ of the water at 5 ft intervals from one end of the pool to the other. Estimate the volume of water in the pool where $V = \int_0^{50} 30\, h(x)\, dx$

Using:

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i) Trapezoidal rule with $n = 10$ (3 marks)

ii) Simpson's Rule with $n = 10$ (3 marks)

Position f (t)	X	0	5	10	15	20	25	30	35	40	45	50
Depths (ft)	Y	6	8.2	9.1	9.9	10.5	11.0	11.5	11.9	12.3	12.7	13.0

Where y = 30 h (x)

b) Estimate the absolute value of the maximum error that can occur when approximating $\int_1^4 \frac{dx}{x}$ by the trapezoidal rule with $n = 20$ and Simpson's Rule with $n = 20$ (8 marks)

c) Find the coordinates of the centroid (centre of mass) of the first quadrant arc of the circle $x^2 + y^2 = a^2$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate the following integrals

i) $\int \frac{1+2x^2}{x^5(1+x^2)^3}\,dx$ use the substitution $u = x^4 + x^2$ (5 marks)

ii) $\int \frac{1}{\sqrt{2x - x^2}}\,dx$ (4 marks)

iii) $\int \frac{x^2 + 2x}{(x+1)^2}\,dx$ (5 marks)

b) How many subdivisions, n, should be taken to ensure that the absolute value of the error in the approximation of $\int_1^4 \frac{dx}{x}$ is at most 0.0001 when using

i) Simpson's Rule (3 marks)

ii) Trapezoidal Rule (3 marks)