



KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2009/2010

INSTITUTE OF OPEN LEARNING EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF ARTS

SMA 200: CALCULUS II

DATE: Monday 15th February, 2010

TIME: 11.00 a.m. – 1.00 p.m.

INSTRUCTIONS

Answer question ONE and any other TWO questions.

Question one

a) Evaluate the following integrals

i) $\int 6x^2(2x^3 + 1)^{10} dx$ [2 marks]

ii) $\int \cos 5x \cos 3x dx$ [2 marks]

iii) $\int_2^3 \frac{dx}{(x-1)^2(x^2+1)}$ [4 marks]

b) Find the length of the arc of the curve $x = t^2, y = t^3$ from $t = 0$ to $t = 4$
[5 marks]

c) Find the area enclosed by the curve
 $y = -x^2 + 3x + 10$ and the line $y = 2x + 4$ [4 marks]

d) Find the volume generated when the area enclosed by $y = 3x - x^3$ and the x-axis is
rotated about the x-axis. [5 marks]

f) Suppose it is found experimentally that two physical variables x and y are related
as shown in the table below

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	3.1	4.0	4.2	3.8	2.9	2.8	2.7

Taking $y = f(x)$ where f is continuous, evaluate $\int_1^4 f(x) dx$ using Trapezoidal and Simpson's rules. [5 marks]

- g) Determine whether the improper integral below diverges or converges

$$\int_0^3 \frac{dx}{(dx-1)^{2/3}} \quad [4 \text{ marks}]$$

Question Two

- a) Express $\frac{4x^2 - 3x - 4}{x^3 + x^2 - 2x}$ into partial fractions. Hence evaluate

$$\int_0^4 \frac{4x^2 - 3x - 4}{x^3 + x^2 - 2} dx$$

- b) Evaluate $\int_0^{\pi/4} \frac{dx}{1 + \sin 2x}$

(Use $t = \tan x$)

[7 marks]

- c) Use integration by parts to evaluate

$$\int x^3 e^{-x} dx$$

[5 marks]

Question Three

- a) Given that $6xy = 3 + x^4$ show that $1 + \left(\frac{dy}{dx}\right)^2 = \left[\frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)\right]^2$

Hence find the length of the arc of the given curve between the points

$x = 1$ to $x = 4$.

[7 marks]

- b) The curve $3y = (3 - x)\sqrt{x}$ between $x = 0$ and $x = 3$ is rotated about the x -axis.

Find the surface area of the solid formed.

[7 marks]

- c) Given the line $y = 3x + 2$ find the volume of the solid of revolution generated by the line and the x -axis from $x = 1$ to $x = 3$.

[6 marks]

Question Four

a) Tabulate the 4 decimal places the values of the function $f(x) = \frac{4}{1+x^2}$ for the values of x from 0 to 1 with n = 6. [4 marks]

b) Use the values in (a) above to estimate the value of pi using

- i) Trapezoidal rule
- ii) Simpson's rule

where $\pi = \int_0^1 \frac{4}{1+x^2} dx$ [6 marks]

c) Use integration to evaluate $\int_0^1 \frac{4}{1+x^2} dx$ to 4 decimal places. Hence find the percentage error in using each of the rules in (b) above. [6 marks]

d) Find the indefinite integral

$\int (x^2 - 1)\sqrt{x^3 - 3x + 1} dx$ [4 marks]

Question Five

a) i) If $I_n = \int \cot^n x dx$ prove that $I_n = \frac{1}{(n-1)} - I_n - 2 (n \geq 2)$

ii) Use the above reduction formula to evaluate

$\int_0^{\pi/4} \cot^5 x dx$ [10 marks]

b) Evaluate the integral $\int x^3 \cos x dx$ [4 marks]

c) Investigate the convergence or divergence of the improper integrals

i) $\int_1^{\infty} \frac{dx}{x+4}$ [3 marks]

ii) $\int_0^2 \frac{dx}{\sqrt{x(2-x)}}$ [3 marks]

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