

Mount Kenya



University

UNIVERSITY EXAMINATION 2012/2013

**SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF NATURAL SCIENCES**

**BACHELOR OF EDUCATION SCIENCE
SCHOOL BASED**

**UNIT CODE: ECS 211 UNIT TITLE: BASIC DISCRETE
MATHEMATICS**

DATE: AUGUST 2013 MAIN EXAM TIME: 2 HOURS

INSTRUCTIONS:

Answer ALL questions in section A and answer ANY TWO questions from Section B

SECTION A: COMPULSORY

1.
 - a. What is a proposition?
 - b. If x and y are Boolean algebra, complete the following multiplication and addition table below. (4 Marks)

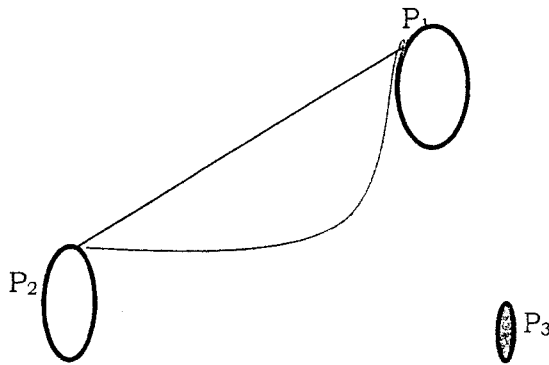
x	y	$X+y$	$x.y$
1	1		
1	0		
0	1		
0	0		

- c. Suppose a language contains only one object, student and two relation constants, slow learner and quick learner. If M is a model, state the meaning of the following models:
- i. $M = \emptyset$
 - ii. $M = \{\text{Slowerlearner}(\text{student})\}$
 - iii. $M = \{\text{Slowlearner}(\text{student}), \text{quicklearn}(\text{student})\}$
- d. Show that if p and q are propositions then $(\overline{p \vee q})$ and $\overline{p} \wedge \overline{q}$ are logically equivalent using the truth table.
- e. If the universal set U is the set $\{0, 1, 2, 5, 7, 10\}$ and the subsets of U are defined as:
- $A = \{1, 2, 5, 10\}$
- $B = \{0, 1, 7, 10\}$
- $C = \{0, 1, 2, 5\}$
- i. Draw a Venn diagram to represent the relationship between U, A, B and C . (4 Marks)
 - ii. Using the Venn diagram or otherwise find the members of sets $A \cap B \cap C$ and $A \cup B \cup C$. (2 Marks)
- f. Given the six lettered word "square"
- i. How many ways can this word be permuted? (2 Marks)
 - ii. In how many of these ways r is the second letter? (2 Marks)
 - iii. How many of these ways are q and e next to each other? (2 Marks)
- g. If n is any natural number prove by mathematical induction the formula:

$$2 + 4 + 6 + 8 + \dots + 2n = (n+1).$$

(3 Marks)

h. Write the matrix representing the following graph.



SECTION B:

2.

a. Eighty-two individuals have complained to the consumer protection Agency about a car make. The information contained in the complaint letters is summarized below:

- i. 25 complained about steering
- ii. 23 complained about comfort
- iii. 22 Complained about visibility
- iv. 11 complained about steering and comfort
- v. 7 complained about steering and visibility
- vi. 5 complained about all the three
- vii. 33 complained about none of these

How many people complained about comfort and visibility but not steering? (8 Marks)

b. Prove by mathematical induction that the sum of the geometric progression.

$$a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = \frac{a - ar^n}{1 - r} \quad r \neq 1$$

For all natural numbers n. (6 Marks)

c. i. Evaluate ${}^{20}C_2$ (3 Marks)

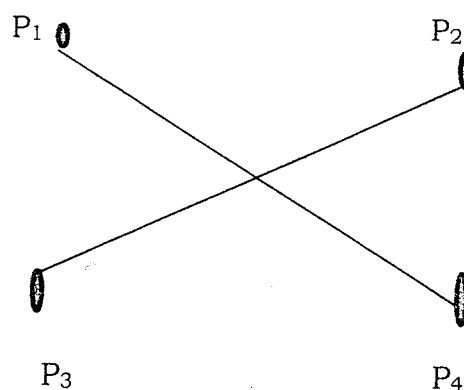
ii. In how many different orders can 10 mathematics books be shelved if each set consists of 3 books? (3 Marks)

3.

a. i. What is a tautology? (2 Marks)

ii. If A and B are propositions prove that $A \wedge \bar{b} \rightarrow (\overline{A \rightarrow B})$ is a tautology using the truth table. (6 Marks)

b. i. Represent the following graph using the adjacency matrix. (4 Marks)



ii. Draw the picture of the graph that has the following matrix.

(2 Marks)

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c. i. Evaluate ${}^{15}P_3$

(3 Marks)

ii. Find how many different numbers can be made using four of the nine digits 1, 2, 3, ..., 9.

(3 Marks)

4.

a. Given that x and y are Boolean algebra, prove one of the De Morgan's law in the network terms: $\text{Not}(x \text{ and } y) = (\text{not } x \text{ or } \text{not } y)$ by completing the table below.

(5 Marks)

x	Y	Notx	Noty	(x and y)	Not (x and y)	Notx or noty
1	1					
1	0					
0	1					
0	0					

b. Three sets A, B and C are defined recursively as:

i. A is a set that:

a. $3 \in A$

b. If $x \in A$, then $2x \in B$

ii. B is a set such that:

a. $1 \in B$

b. If $x \in B$ then $2x \in B$

iii. C is a set such that:

- a. $0 \in C$
- b. if $x \in C$, then $x+1 \in C$
- c.

a. Write first five members of each set. (3 Marks)

b. Find the sets $A \cap B$, $A \cup B$ and $A \cap (B \cup C)$ (3 Marks)

c. i. What is a loop? (2 Marks)

ii. What is a digraph? (2 Marks)

iii. The following matrix represents a digraph. Use it to draw the digraph. (5 Marks)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

5.

a. Use mathematical induction to prove the formula

$$3+6+9+\dots+3n = \frac{3n(n+1)}{2} \text{ if } n \text{ is any natural number. (5 Marks)}$$

b. If p and q are propositions prove that $(\bar{p} \vee q) \wedge (p \wedge \bar{q})$ is a contradiction. (5 Marks)

c. Nine players are available to play for a tennis team of 4 players. In how many ways the team can be selected if 2 players are brothers and must both be included in the team. (4 Marks)

d. Data including geographical location, city size and marital status, on 200 recent graduates of gigantic state University are collected by Allumini Association. The results were as follows:

108 Live in the west

86 Live in a large city

68 are married.

41 Live in large city in the west

23 are married and live in large city

19 are married and live n the west

12 are married and live in large city in the west.

How many unmarried, do not live in large city and do not live on the west?

(6 Marks)