

Mount Kenya



University

UNIVERSITY EXAMINATION 2014/2015

SCHOOL OF PURE AND APPLIED SCIENCES  
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

BEDA/BEDS/BSNE/BENH  
SCHOOL BASED

UNIT CODE: BMA1102

UNIT TITLE: CALCULUS I

DATE: JUNE 2015

SUPP / SPECIAL EXAM

TIME: 2 HOURS

**INSTRUCTIONS:** Answer questions one and any other two questions.

1. a) Evaluate;  $\lim_{x \rightarrow 3} \frac{x^4 - 81}{x - 3}$  (4 Marks)
- b) Find the values of a and b that make f(x) continuous on the entire real number line.  $f(x) = \begin{cases} 2x & \text{if } x < 1 \\ cx^2 + d & \text{if } 1 \leq x \leq 2 \\ 4x & \text{if } x > 2 \end{cases}$  (5 Marks)
- c) A small spherical balloon is inflated at the rate of 2 cm<sup>3</sup>/s
- i) What is the rate of growth of the radius? (3 Marks)
- ii) Find the rate increase of the radius when the volume of the balloon is 50cm<sup>3</sup> (4 Marks)
- d) Given  $y = \frac{t^2 + 2}{t^3 - 4t}$  find  $\frac{dy}{dx}$  (5 Marks)
- ii) Differentiate  $y = \sin(xy)$  implicitly (4 Marks)

e) The parametric equations of a curve are  $x=e^t$ ,  $y=\sin t$ . Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  (5 Marks)

2. a) Find the equation of the tangent and the normal lines to the curve.

$$x^2y^2 - 2x = 4 - 4y \text{ At the point } (2, -2) \quad (6 \text{ Marks})$$

b) Find the maximum and minimum value of the function.  $y = 2 \sin t + \cos 2t$  (6 Marks)

c) A body moves along a straight line according to the law.

$$s = t^3 - 6t^2 + 9t + 4$$

Find:

i)  $s$  and  $a$  when  $v=0$  (5 Marks)

ii)  $s$  and  $v$  when  $a=0$  (3 Marks)

3. a) A farmer has an adjustable electric fence that is 100m long. He uses this fence to enclose a rectangular grazing area on three sides, the fourth side being a fixed hedge. Find the minimum area he can enclose. (6 Marks)

b) Find the derivatives of;

i)  $y = \tanh x$  (4 Marks)

ii)  $y = \operatorname{sech} x$  (3 Marks)

c) Find  $\frac{dy}{dx}$  in the following;

i)  $y = \coth^{-1} \sqrt{x^2 + 1}$  (4 Marks)

ii)  $y = \ln(\tanh^{-1} x)$  (3 Marks)

4. a) Differentiate the following using the rule indicated in brackets.

i)  $y = \cos^4 \left( 2\theta - \frac{\pi}{5} \right)$  (Chainrule) (5 Marks)

ii)  $y = 12x^3 + 2x^2$  (first principles) (5 Marks)

iii)  $y = (1 - 6x^3)(4x^2 - 6x + 2)$  (product rule) (5 Marks)

iv)  $y = \frac{x^2 - 1}{x^2 + 1}$  (Quotient rule) (5 Marks)

5. a) Evaluate;

i)  $\lim_{x \rightarrow \infty} \left( \frac{x^4 - x + 4}{3x^4 + x^3 + 3} \right)$  (3 Marks)

ii)  $\lim_{x \rightarrow 2} \left( \frac{x^3 - 8}{x^2 - 4} \right)$  (3 Marks)

b) Discuss the continuity of;

i)  $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ \frac{x^2 - x - 2}{x^2 - 4} & \text{if } x = 2 \end{cases}$  (3 Marks)

ii)  $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{for } x < 2 \\ \frac{x^2 - x - 2}{x^2 - 4} & \text{for } x > 2 \end{cases}$  (3 Marks)

c) Differentiate the following implicitly with respect to x.

$$x^2 y^3 - xy^2 = 10xy \quad (4 \text{ Marks})$$

d) Differentiate;  $y = \log_7 \tan^3 x$  (4 Marks)